Circular motion

November 22, 2025

1 Background

Newton's laws give us the equations of motion for classical bodies. However, some motions in particluar are so important that they are worth additional study. One of those cases is circular motion, the topic of this handout. First,we will derive the necessary conditions for circular motion to occur. The rest of the handout will be about applying this condition to calculate various properties of known circular motions, e.g. the time period, the radius, the centripetal force.

Consider a body that moves with a velocity $\mathbf{v}$ in the vicinity of another body. Let's imagine that it's a planet orbiting a star which is fixed in place. If there was no force on the moving planet, what would happen to it? That's right, by Newton's first law, it would just ocntinue moving with its current velocity, until the end of time. So, there would be no circular motion. That means that there must be a force on the planet for it to begin orbiting the star. Remember, a force is a vector, so to determine the force we must tdetermine both its magnitude and its direction. Let's start with the direction: can you figure out in what direction the force must be?

If we consider how the velocity must change for the planet to orbit the sun, we see that it must turn toward the star (to the left, in the figure). This means that the acceleration of the planet must be inward, toward the sun. Furthermore, for the planet to orbit in a perfect circle, that means that the motion must be completely radially symmetric around the star. This means that the speed must be constant. In other words, no work can be done on the planet, and thus the force must be completely perpendicular to the planet. Thus, we get a force diagram like that shown in figure **??**.

So, we have figured out the direction of the force. Now, we just want to calculate the magnitude and we'll be done. In our session, I will show how this is done, and we will arrive at the classic results:;

$$F = \frac{mv^2}{R} = m\omega^2 R$$

where m is the mass of the of the orbiting planet, v is its speed, and R is the radius of the orbit (and $\omega = v/R$ is the angular speed, which is sometimes more convenient to use).

If you want to see a derivation of this but missed our session, you can check out chapter 4.5 In Physics by Halliday, Resnick, & Krane. You can find the book online here.

2 Questions

1. A car of mass $m = 1500$ kg travels around a flat, circular curve with a radius of $R = 50$ m. If the car maintains a constant speed of $v = 20$ m/s, calculate the magnitude of the centripetal force required to keep the car moving in this circular path.

2. A ball of mass $m = 0.2$ kg is attached to a string of length $R = 0.5$ m and spun in a horizontal circle on a frictionless table. The string will break if the tension exceeds $F_{max} = 16$ N. Using the centripetal force formula, calculate the maximum speed v the ball can reach before the string breaks.

3. A satellite is to be placed in a circular *geostationary* orbit around the Earth. Use the constants

$$M_E = 5.97 \times 10^{24}\,\text{kg}, \quad G = 6.67 \times 10^{-11}\,\text{N}\,\text{m}^2\text{kg}^{-2}$$

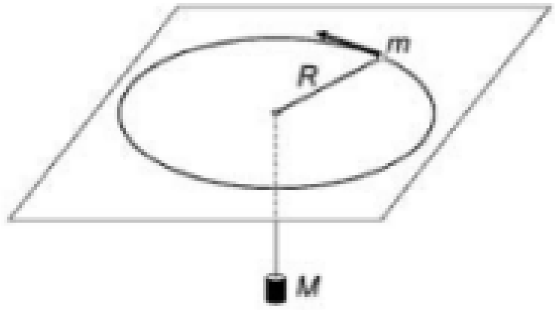

Figure 1: Falling mass on string

(Earth's mass, gravitational constant).

Derive an expression for the orbital radius R of a geostationary orbit and calculate its numerical value.

4. A motorcycle rides around the inside of a vertical cylinder of radius R. So that the motorcycle does not tip over, it must make an angle with the normal directed out from the wall. What is this angle if the speed of the motorcycle is v? Assume that the friction between the tires and the wall is sufficient so that it does not slip, and that the height of the motorcycle with rider is much smaller than the radius of the cylinder.

5. The International Space Station (ISS) orbits the Earth in an approximately circular orbit at an altitude of 400 km above the Earth's surface. The total mass of the station is 4.2×10^5 kg. At this height space is not completely empty. The ISS is therefore subjected to a small frictional force such that the orbital altitude is reduced by about 2 km per month if the orbit is not actively adjusted. Find an estimate of the frictional force.

6. A ball of mass m is attached to a string of length L, with its other point fixed. If the string starts out hanging down vertically, what is the minimum starting speed of the ball v for the ball to move in a vertical circular motion around the string's fixed point?

7. The Russian Pavel Kulizhnikov set a new world record in the 500 m speed skating at a World Cup event in Salt Lake City on 20 November 2015. The new time is 33.98 s. He skated the last outer curve, which has a minimum inner radius of 30.0 m. Towards the end of the curve he is in the middle of the lane and his speed is measured to be 61.0 km/h. Each lane has a width of 4.0 m. We assume that Pavel has a mass of 85 kg.

   What is the force from the surface when only one skate is in contact with the ice, and what angle does his body make with the ice?

8. A car is driving in a roundabout. A ball is suspended from the ceiling of the car by a light string, and the ball hangs at an angle of 30° from the vertical direction. The car has a constant speed of 72 km/h. Determine the diameter of the roundabout, i.e. the diameter of the circle that the car follows.

9. A cart of mass M has the shape shown in figure 2. The surface is frictionless, with a horizontal part and a quarter circle of radius R. The cart can roll freely without friction along a horizontal track. A block of mass m slides onto the cart with speed v. Assume that the speed is large enough that the block leaves the cart at the top of the quarter circle. What is the highest point in the block's trajectory?

10. Two blocks, A of mass m and B of mass $2m$, are released simultaneously from the top of a semicircular frictionless track of radius r; see the figure. They collide in a completely inelastic collision at the lowest

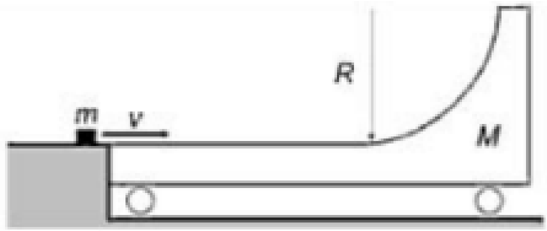

Figure 2: Mass onto trolley

Figure 3: Masses on semi-circle

point of the track. Find an expression for the force from the track on the combined body immediately after the collision.

11. A ball of mass $m = 1.0\,\mathrm{kg}$ is attached to a string and moves in a circle of radius $R = 40\,\mathrm{cm}$ on a horizontal surface (see figure 1). The string passes through a hole at the centre of the circle, and at the end of the string hangs a weight of mass $M = 2.0\,\mathrm{kg}$. If the weight is released, the ball will at its closest be at a distance $r = 10\,\mathrm{cm}$ from the centre. Neglect friction everywhere.

    a) Find the largest and the smallest speed of the ball.
    b) Find the acceleration of the weight at the highest and lowest point of its motion.

12. We want to store electrons by letting them move in circular orbits in a magnetic field. The electrons are to move in a horizontal circle with its centre on the z-axis, and they are to move with speed v. The electrons have charge q and mass m. If the magnetic field is homogeneous, the electrons will fall down because of gravity. We therefore use a magnetic field with a component in the z-direction given by

$$B_z = B_0 \left(1 - \frac{z}{z_m}\right),$$

and a component in the radial direction given by

$$B_r = \frac{B_0}{2z_m}\, r,$$

where B_0 and z_m are given constants.

At what height z must we send the electrons in so that they stay at the same height, and what must the radius of the circle be?

13. A small ball is placed at rest on the top of a fixed, smooth (frictionless) larger sphere. It is given a tiny push and starts to slide down the surface. Let θ be the angle between the vertical line through the centre of the sphere and the radius to the ball (i.e. the angle the radius has rotated from the "top" position). For what angle θ does the ball lose contact with the sphere?

Answers

1. $12,000$ N (or 12 kN)
2. 6.32 m/s
3. $R \approx 4.22 \times 10^7$ m (or $42,200$ km)
4. $\theta = \arctan\left(\frac{Rg}{v^2}\right)$
5. 0.18 N
6. $v = \sqrt{5gL}$
7. $F \approx 1130$ N, $\theta \approx 47.5°$
8. 142 m
9. $H = \frac{Mv^2}{2g(M+m)}$ (height above the initial horizontal track)
10. $F = \frac{11}{3}mg$
11. a) 3.54 m/s and 0.89 m/s; b) 5.88 m/s^2 and 35.2 m/s^2
12. $r = \frac{2z_m mg}{qvB_0}$ and $z = z_m - \frac{v^2}{2g}$
13. $\theta = \arccos(2/3) \approx 48.2°$